

# The Physical Interpretation of Special Relativity – a Vindication of Hendrik Lorentz

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Modern astronomy has revealed the existence of a cosmologically-based fundamental reference frame associated with the distribution of matter in our universe. The existence of such a frame offers a firm basis to H. A. Lorentz's approach to the understanding of the relativistic effects associated with the transformation named after him.

Lorentz's approach and his notion of an "ether" evolved over a number of years, and it has been further refined as a result of our new view of the universe. It now provides a complete interpretation of Special Relativity based on a single assumption – that light-propagation takes place with respect to a unique, observable, fundamental reference frame. This interpretation links Einstein's theory with Lorentz's findings and so further develops our understanding of it; it also offers new insights for the better understanding of quantum and particle physics.

## 1. The Nature and Representation of Einstein's Approach

Hendrik Lorentz believed to the end of his days that his interpretation of relativistic effects and null-effect phenomena (e.g. as manifested by the Michelson-Morley experiments), in terms of the existence of a unique reference frame associated with an aether, had some advantages over Einstein's approach. He gave due credit to the greater completeness of Einstein's theory which, he admitted, is mathematically simpler than his own but at the cost of submerging the physical basis of the theory so that "Einstein simply postulates what we have deduced, with some difficulty and not altogether satisfactorily, from the fundamental equations of the electromagnetic field" [1].

Einstein's approach had other advantages, perhaps not fully appreciated by Lorentz. Einstein offered a precise operational meaning for the coordinates employed in the Lorentz transformation. Without Einstein's measurement conventions in terms of reflecting light-signals, the full significance of the Lorentz transformation and its consequences – for instance, the "relativity of simultaneity" and the reciprocity of observations between observers in different inertial frames – cannot be fully comprehended, even in terms of Lorentz's physically-based approach; this is one of

the reasons why Lorentz's approach was considered, even by himself, to be incomplete.

Einstein's operational emphasis implies, indeed, the hidden existence of an *actual* "stationary frame", a notion employed in many contexts of his 1905 paper [2]. Thus, in dealing with the length of a "moving" rod according to 'stationary' observers, he uses  $c-v$  and  $c+v$  as the speeds of light in the two directions relative to the moving rod. He goes on to explain that this means "that we cannot attach any absolute significance to the concept of simultaneity" [2], since clocks synchronous (according to Einstein's light-signal convention which requires the speed of light to be taken as  $c$  in both directions) in the stationary system would not appear so to observers and the Einstein-synchronous clocks associated with the moving rod system, and vice-versa. Thus Einstein actually realised that his measurement convention introduced a synchronism discrepancy effect which underlies 'the relativity of simultaneity'.

Further, in his deduction of the Lorentz transformation, Einstein employs  $c-v$  and  $c+v$  freely as the light-speeds relative to the moving system. Thus he employs a Lorentzian approach to the understanding and deduction of his results in spite of his denial of the actual existence of his "stationary frame" which makes these results physically intelligible.

The subsequent re-expression of Special Relativity by Minkowski in terms of the invariance of the space-time metric

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2,$$

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removed all the physical content and light kinematics from Einstein's theory, and this is the way the theory has been considered and taught ever since. In this way did Einstein's theory and its presentation evolve after 1905, but it is important to realise that the theory evolved from long before 1905, and many scientists were involved in this evolution.

## 2. The Beginnings of Relativity Theory

The first of these was Voigt [3] who, in 1887, proposed a set of Lorentz-type transformations which might apply better to Maxwell's equations than does the Galilean transformation.

Next, Heaviside [4] deduced in 1889 a retarded potential result for the field of a charged particle moving at speed  $u$  through a dielectric. This result implied a compression of the (co-moving) field in the direction of motion, and a resultant contraction of the particle and of a system of particles by the factor,  $(1 - u^2/c^2)^{1/2} \equiv \beta^{-1}$ , in the direction of motion. The implications of this result were immediately appreciated by Lodge and Fitzgerald, the latter being the first to recognise its great significance for making the null-results of the Michelson-Morley experiments physically intelligible. Thus in a letter to Science he wrote "We know (following Heaviside's result – S.J.P.) that electric forces are affected by the motion of electrified bodies relative to the ether and it seems a not improbable supposition that the molecular forces are affected by the motion and that the size of the body alters consequently" [5], thereby explaining the Michelson-Morley null-effects without resort to the notion that the ether is carried along by the Earth. So, as Bell [6] emphasises, Fitzgerald did not simply offer an arbitrary hypothesis to explain the null-results; he showed that this hypothesis was reasonable because, following Heaviside, electrical forces change with motion, and he considered that molecular forces might behave in a similar way.

Later on, in his Unpublished Papers [7], Heaviside further proposed that the retarded potential effect extends to gravitational fields, so that a system of neutral particles would be similarly contracted in the direction of motion. This was a brilliant insight, but it could only be properly justified as a well-based hypothesis if, following Einstein's general relativistic notion of gravitational waves, the transmission of gravitational energy could be treated as having the same limiting speed as other forms of energy in the

context of an observable unique reference frame (revealed by modern astronomy) in respect to which all forms of energy propagate.

Quite independently Hendrik Lorentz [8] offered the same contraction hypothesis in 1892, explaining that this might well be due to the similar behaviour of molecular and electrical forces, knowing that the latter are affected by the Heaviside retarded potential result for moving charges. He presented this argument more fully in his 1894 article [9] in which he also acknowledged the priority of this argument to Fitzgerald and Lodge.

Lorentz continued to develop his theory of electromagnetic phenomena in moving systems and, independently, so did Joseph Larmor. As early as 1897 Larmor [10] deduced anew the retarded potential field effect and the resulting length contraction by the factor  $\beta^{-1}$  for a system of charged particles; he then showed that electron-associated phenomena must then operate more slowly by the factor  $\beta$  in moving systems than in stationary ones; and, finally, he deduced the synchronisation discrepancy effect,  $vx/c^2$ , for a distance  $x$  (in the direction of motion) relating to "the difference in time reckoning" between the two systems. The latter is, of course, the source of "the relativity of simultaneity", a second 'time effect' resulting from motion relative to a stationary frame, and the interaction of these two effects produces the reciprocity of observations between observers in the two frames and the observational equivalence between them. However, it was left to Lorentz [11] to describe the transformation which describes the interaction of all these effects.

Lorentz had been working for many years towards a transformation (of the different measures associated with two different inertial frames) which would render Maxwell's equations invariant in respect to all inertial frames. By 1904 [11] he had achieved this and was also able to deduce from his transformation the length-contraction and mass-velocity dependence formulae, and other new results of electromagnetic theory.

Meanwhile, Henri Poincaré had been watching and commenting on these developments with great interest. In 1900 he proposed [12] an 'impotence' Principle of Relativity: 'In consequence of a generalised action-reaction principle operating in the physical world, movement relative to a hypothetical ether frame is not detectable', so that all inertial frames are thereby observationally equivalent. He summed up his conclusions [13] in 1905: enunciated a Light Principle ('The

speed of light is a limiting speed'), expressed what he called "the Lorentz transformation" in its final and most satisfactory form, and pointed to the group properties of this transformation which represent the mathematical counterpart of the equivalence of inertial frames and the reciprocity of observations between such frames.

Thus by June, 1905, the essential bases of Special Relativity and its physical interpretation had been developed.

### Einstein's Contribution

In July, 1905, Einstein's own contribution [2] to the problem appeared, and it certainly advanced the theory and its implications a great deal. He treated the observational evidence as his basic assumptions, his Relativity and Light Principles, and took the signal step of defining what he meant by the distance and time *co-ordinates* of an event or body by proposing operational *conventions* which employed reflecting light-rays and clocks. He proposed a similar convention for synchronous clocks in a given inertial frame.

He then presented, for the first time, a systematic deduction, from his assumptions and definitions (of the co-ordinates, etc.), of the Lorentz transformation, thus demonstrating that it was his conventionally-measured co-ordinates which were related by the transformation.

He deduced, from the transformation, the length-contraction and time-dilation results, and proposed that the latter was an absolute effect for out-and-return journeys and should be detectable by comparing similar clocks at the equator and poles of the Earth. (This effect, associated with the Earth's rotation, was indeed confirmed in an ingenious experiment by Hafele and Keating [14] 67 years later.)

He deduced a new composition of velocities formula, confirmed the invariance of Maxwell's equations under a Lorentz transformation, and proposed audaciously to modify all the laws of mechanics as well as of electrodynamics and optics to make them similarly invariant. This led directly to a demonstration of mass-velocity dependence, and to a mass-energy equivalence relationship.

It is seen that Einstein's contribution heralded a veritable revolution in our ways of observing nature and describing the physical laws of nature, and it had

a number of other consequences. His view of co-ordinates as necessarily based on conventionally-defined light-signal measurements encouraged an operational, philosophically-positivist approach to all physical science. It was interpreted as meaning (contrary to Einstein's own views) that science should only deal with observations, and that laws can only relate such observations – that we should not attempt to try to understand what lies behind the observations, that such attempts are futile and meaningless.

It was in this operationalist climate that quantum theory developed and blossomed this century. Einstein, who had made important contributions to this theory, was aghast at the quantum theorists who refused to recognise any objective reality behind the observations and notions of quantum mechanics, and the controversy around the physical meaning (if any!) of the quantum wave-function, etc., has continued long after Einstein's frequent challenges to Bohr and his "Copenhagen interpretation".

Einstein's approach involved the theoretical development of a problem which had been studied by his predecessors in terms of the physical consequences of motion relative to an ether – a notion that Einstein dismissed as unobservable and unnecessary. Further, as we have seen, Einstein's approach was completely geometrised by Minkowski [15], and its physical content was thereby further concealed and neglected. In the ensuing positivist climate, attempts at realist physical interpretations were discouraged and derided – it was sufficient that a theory was mathematically self-consistent and 'worked'. So in this climate, the physical interpretation of relativistic effects developed by Lorentz and others was increasingly neglected and treated as redundant.

Yet there has always remained a resistance to Einstein's approach. His deduction of an absolute time-dilation effect, which is a function of the speed of a clock relative to a given inertial frame, has generated a recurrent controversy around the so-called "clock paradox", a paradox which apparently ignores the reciprocity of observations between 'moving' and 'stationary' observers. This (and other) apparent ambiguities in Einstein's theory is due, according to its critics, to its lack of physical basis; though, to be fair, the ambiguities are only apparent from the viewpoint of the pure theorists who claim that the theory and its results are perfectly self-consistent mathematically. Nevertheless, an approach which might dissolve the ambiguities could be considered preferable.



### The Cosmological Imperative

Lorentz appreciated the power of Einstein's approach and admitted the incompleteness of his own. However, to the end of his life he believed in the heuristic and explanatory advantages of assuming the existence of a luminiferous ether. The notions about such an ether had also undergone considerable evolution throughout the 19th century. By 1909 it sufficed for Lorentz [1] that the ether be considered as the seat of electric and magnetic fields, and he treated it as merely a unique inertial frame in respect to which light travels at speed  $c$  in all directions. Einstein's Light Principle and the equivalence of inertial frames could then be deduced from the existence of such an ether. This also explained why motion relative to the ether or the effects of such motion could not be observed – a distinct handicap to the Lorentzian approach and an apparently decisive advantage for Einstein's.

However, around 1930 the situation changed: our notion of the universe was completely altered by Hubble's astronomical revelation that the universe should be considered as an (expanding) ensemble of galaxies distributed fairly homogeneously on the large scale as far and wide as our telescopic vision stretches. Since 1930 this view of the universe has been vindicated and greatly amplified, and we can treat our universe of galaxies as associated with a unique fundamental reference frame which, following the discovery of Penzias and Wilson [16] is also associated with a universal microwave background radiation of temperature 2.7 K. So, we can now actually estimate our local motion relative to the universe at large by measuring the temperature of the background radiation in different directions from our standpoint; it appears to be about 300 kilometres per second.

Thus, whether we like it or not, we do now have an observable 'preferred' reference frame, and it may be shown [17] from the Robertson-Walker theory describing this expanding frame that it is the unique frame in respect to which light (all forms of energy) propagates at speed  $c$  in all directions. The manifest existence of such a frame constitutes a cosmological imperative; yet strangely, this is ignored and evaded by the main body of physicists who still insist that the Einstein-Minkowski version of Special Relativity is sufficient, as if no preferred frame existed! Further, the endeavours of particle physicists has led to the conviction that empty space is by no means featureless but, instead, should be considered as a *physical* vacuum – a likely seat for field activity as envisaged by Lorentz.

In the new context of an observable and defined fundamental reference frame for light propagation, the physical theory developed by Heaviside, Fitzgerald, Larmor, Poincaré and Lorentz (and others) falls into place very beautifully, and their programme has been further systematised and generalised by Ives [18], Builder [19] and Prokhovnik [20]. The 'Neo-Lorentzian' approach to Special Relativity resolves all the ambiguities and apparent paradoxes of the theory [21].

Of course, Einstein's innovative notions (for example, the measurement meaning of space and time co-ordinates and the consequences of his measurement conventions), have proved tremendously important for the completion of Lorentz's approach. Thus the linking of Einstein's theory and Lorentz's interpretation amplifies both the theory and its physical meaning, and provides an expression of Special Relativity based on a *single* assumption of a cosmologically-based fundamental reference frame. In this new form the theory and its fundamental frame basis also offers new insights into quantum physics and quantum electrodynamics [17].

Certainly, Lorentz, and his co-workers are fully vindicated by the findings of modern cosmology and particle physics, and so is Einstein – his theory remains entirely valid but becomes inestimably richer and more intelligible through its Lorentzian interpretation.

### Appendix: The Consequences of Movement Relative to the Fundamental Frame

The existence of a fundamental reference frame for light propagation means that the speed of light is precisely isotropic with respect to any body (or fundamental particle or associated fundamental observer) which is stationary in respect to this frame,  $I$ . It follows, then, that the speed of light will not be isotropic with respect to a body or observer (or their associated reference frame) moving, with velocity  $u$  (say), relative to  $I$ . It is easily seen that, in respect to such a body, the speed of light approaching the body will be as shown in Fig. 1, where

$$c' = (c^2 - u^2 \sin^2 \theta)^{1/2} + u \cos \theta \quad (1)$$

for the direction making an angle  $\theta$  with the direction of  $u$ . We may call the result (1) the *primary anisotropy effect* due to motion relative to  $I$ .

A second consequence of such movement results from the retarded potential effect on the fields (gravi-

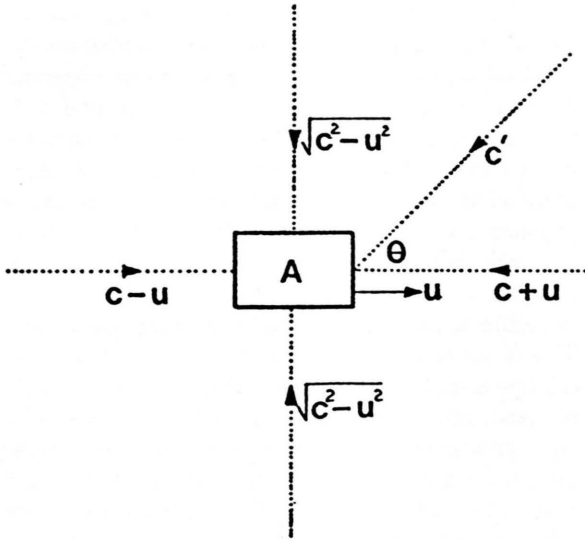


Fig. 1. Speed of light relative to a body moving with velocity  $u$  with respect to the fundamental frame  $I$ .

tational and/or electromagnetic) associated with the moving body and its constituent particles. Following Einstein's view of gravitational fields and in the context of the existence of a frame such as  $I$ , the variation of the potential at any point of a gravitational field, due to the movement of its source, will involve a time-lag depending on the distance of the point from the source, exactly as is well-known for the electromagnetic field of a moving body. It follows [20] that the usual inverse-square law for such a field is then modified by a factor involving the body's speed  $u$  relative to  $I$ , so that

$$F = \frac{-GmM}{r^2} \frac{(1-u^2/c^2)}{[1-(u^2/c^2)\sin^2\theta]^{3/2}}. \quad (2)$$

for any direction making an angle  $\theta$  with the direction of  $u$ , and where  $m$  is the rest-mass of a test particle, stationary in  $I$ <sup>1</sup>.

The result (2) implies that the Coulomb and gravitational fields (and any other field of this type), linking a moving system of particles, will be correspondingly asymmetric, so that the system must contract in the direction of motion by the factor,  $(1-u^2/c^2)^{1/2}$ , in order

<sup>1</sup> The effect on a moving test particle involves only a component of the local field effect. Thus in respect to a comoving particle the effect is diminished by the factor  $[1-(u^2/c^2)\sin^2\theta]^{1/2}$  depending on the angle  $\theta$  made by  $u$  with the direction of the field effect at the locality of the particle.

to maintain its internal equilibrium. Hence, a rod of rest-length  $l$  inclined at an angle  $\theta$  to the direction of its velocity  $u$ , relative to  $I$ , will assume a length  $l'$  (according to  $I$  observers), where

$$l' = \frac{l(1-u^2/c^2)^{1/2}}{[1-(u^2/c^2)\sin^2\theta]^{1/2}}. \quad (3)$$

The time-dilation effect now follows as a direct consequence of the interaction of the primary anisotropy and contraction effects. Following Builder [19], consider a light-clock consisting of a rod of rest-length  $l$  with a mirror at each end to reflect a beam of light to and fro along the length of the rod. Let the unit of time be taken as the interval between successive light reflections on one of the mirrors which is connected to a photon-counter. When the rod is stationary in  $I$ , the unit of time,  $\hat{t}$ , is given by

$$\hat{t} = 2l/c.$$

However, when such a clock moves with velocity  $u$  relative to  $I$ , the speed of light relative to the moving rod will, in general, be different for the directions depending on the orientation of the rod-clock relative to the direction of  $u$ . For an angle  $\theta$  to the direction of  $u$ , the two light-speeds,  $c_1$  and  $c_2$ , are given by (1), so that

$$c_1 = (c^2 - u^2 \sin^2 \theta)^{1/2} + u \cos \theta,$$

$$c_2 = (c^2 - u^2 \sin^2 \theta)^{1/2} - u \cos \theta.$$

Hence, the unit of time,  $\hat{t}'$ , for the moving rod-clock is given by

$$\hat{t}' = (l'/c_1) + (l'/c_2) = (2l/c)(1-u^2/c^2)^{-1/2} = \beta \hat{t}. \quad (4)$$

invoking (3) to relate  $l'$  and  $l$ . It is seen that the time-dilation result is independent of the orientation of the light-clock, and that it must also apply to all phenomena involving electromagnetic impulses and energy exchanges. It may be shown that other types of clocks will be similarly affected on account of the anisotropy consequences (1), (2), and (3). Hence Builder contended that this effect will manifest itself not only in physical clocks, but in all natural phenomena, both physical and biological, which have an electromagnetic basis.

It is well known that the contraction effect (3) is sufficient to conceal the light-speed anisotropy to an observer moving uniformly relative to  $I$ : hence the null-result of all Michelson-Morley type experiments. It follows that, in consequence of the time-dilation effect (4), such an observer will further find that his

measure of the speed of light, with respect to his co-moving reference frame, is precisely  $c$  in all directions – as it is for (fundamental) observers stationary in  $I$ . Thus Einstein's Light Principle becomes physically intelligible in our context.

It was in accordance with this Principle that Einstein specified light-signal measurement conventions (which assume light-isotropy) for synchronising clocks and estimating the space and time co-ordinates of distant events, etc. However, it now becomes clear that if observers, associated with a 'moving' reference frame, treat their inertial system<sup>2</sup> as 'stationary' and so synchronise their clocks according to Einstein, then these clocks are bound to appear non-synchronous to an observer in a different frame, and vice-versa. In effect, the employment of Einstein's conventions produces [20] a synchronism discrepancy effect given by

$$(\beta u d / c^2) \cos \theta, \quad (5)$$

where  $d$  is the  $I$  measure of the length-interval separating the (moving) Einstein-synchronous clocks in question, and  $\theta$  is, as usual, the angle made by this interval with the direction of  $\mathbf{u}$ . The result is, of course, equivalent to the relativity of simultaneity factor deduced by Einstein from the Lorentz transformation, and since it is a function of  $\mathbf{u}$  it leads [20] to the relativity of simultaneity in respect to *any* pair of inertial frames in relative motion and hence having different velocities with respect to  $I$ . It is seen that these results demystify Einstein's Light Principle, time dilation and the "relativity of simultaneity".

The anisotropy consequences, (1)–(5), which affect moving bodies and the observations of moving observers,, provide a complete physical interpretation,

free of any ambiguity, of Special Relativity. Their interaction is expressed by the Lorentz transformation. They show how and why the Light Principle operates in respect to all inertial frames, they explain (cf. [22]) why any local experiment designed to detect an absolute velocity is bound to yield a null-effect. The existence of a fundamental reference frame provides a physical basis for these absolute anisotropy effects, and their interaction produces the *local* observational equivalence of all inertial frames in respect to the laws of nature as expressed by the Lorentz transformation. This latter result, a manifestation of the principle of relativity, is by no means fortuitous: it is a consequence of a widely-operating, action-reaction principle proclaimed by Newton, where in this case the anisotropy reactions to uniform motion, relative to  $I$ , nullify precisely the observation by a comoving observer of such motion. Only by astronomical observation can we discern the existence of the fundamental frame and our movement relative to it.

The interpretation, as above, of Special Relativity completes Lorentz's programme for such an interpretation. Lorentz's developing concept of an aether converged towards the notion that it had only the single light-propagation property of our fundamental frame. He was aware of the Heaviside retarded-potential effect which must produce the length-contraction result, but remained to the end rather confused about the nature of time-dilation and about the relativity of simultaneity result which is a direct intelligible consequence of Einstein's measurement conventions. It remained for Builder [19] to disclose the source of these two separate time results and hence present a fully-integrated "Neo-Lorentzian" interpretation of Einstein's Special Theory.

<sup>2</sup> Given that the fundamental frame,  $I$ , is an inertial system, in the sense as described above, then any frame in uniform motion relative to  $I$  at any locality must also be an inertial system.

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- [1] H. A. Lorentz, *The Theory of Electrons* (First edition 1909), Dover, New York 1952.
- [2] A. Einstein, *Ann. Physik* **17**, 891 (1905).
- [3] W. Voigt, *Nachrichten von der K.G.d.W. zu Gottingen* **2**, 41 (1887).
- [4] O. Heaviside, *Phil. Mag.* **27**, 324 (1889).
- [5] G. F. Fitzgerald, *Science* **13**, 390 (1889).
- [6] J. Bell, *Physics World*, Sept. 1992, pp. 31–35.
- [7] O. Heaviside, *Electromagnetic Theory*, vol. 3, Chelsea Publ. Co., New York 1971.
- [8] H. A. Lorentz, *Zitt. d. Akad. v. Wet te Amsterdam*, 1892–1893, p. 74.
- [9] H. A. Lorentz, *Michelson's Interference Experiment* (1894), in: *The Principle of Relativity*, Dover, London 1952.
- [10] J. Larmor, *Phil Trans. Roy. Soc. London* **190**, 205 (1897).
- [11] H. A. Lorentz, *Proc. Acad. Sci. Amsterdam* **4**, 669 (1904).
- [12] H. Poincaré, *Arch. Néerlandaises* **5**, 253 (1900).
- [13] H. Poincaré, *C. R. Acad. Sci. Paris* **140**, 1504 (1905).
- [14] J. Hafele and R. Keating, *Science* **177**, 166 (1972).
- [15] H. Minkowski, *Space and Time* (1908), in: *The Principle of Relativity*, Dover, London 1952.
- [16] A. A. Penzias and R. W. Wilson, *Astrophys. J.* **142**, 419 (1965).
- [17] S. J. Prokhovnik, *Proc. of Trani Conference* (1992), Plenum, New York (in press).
- [18] H. E. Ives, *Phil. Mag.* **36**, 392 (1945).
- [19] G. Builder, *Aust. J. Phys.* **11**, 279; **11**, 457 (1958).
- [20] S. J. Prokhovnik, *Light in Einstein's Universe*, Reidel, Dordrecht 1985.
- [21] S. J. Prokhovnik, *Found. Phys.* **19**, 541 (1989).
- [22] S. J. Prokhovnik, *Found. Phys.* **9**, 883 (1979).